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Equilibrium modeling of beach profile on the Southern Caspian: A case study on Mazandaran coast

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Abstract

In this research, the equilibrium profiles of sediment transport in the Mazandaran coast were investigated using experimental and polynomial models. Bathymetry measurement data as well as existing observations data from some local stations were used to model the equilibrium profiles. According to the results, Dean's mathematical model was in good agreement with the equilibrium profiles of the research area. Also, the polynomial models were more in line with the equilibrium profiles of the study areas compared to Dean's mathematical model and the exponential mathematical models. However, the order of the polynomials varied in different regions which depend on the factors such as different topography, type of bed sediments, and the sediment transfer rate in different seasons. The decreasing slope of the bed from west to east accounted for the difference in the amount of erosion and sedimentation in different coastal areas and, finally, the shape of the bed.

Keywords: Equilibrium beach profile; Mathematical model; Polynomial model; Caspian Sea.

1. Introduction

Beach profiles in different regions are constantly changing under the influence of waves and currents (Kriebel *et al*. 1991). Since the beaches respond to the changes in waves and long-term sea level rise, determining a beach equilibrium profile is useful for interpreting long-term changes in beach profile in response to factors such as long-term increases or decreases in water levels (Larson 1991; Özkan-

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Haller and Brundidge 2007). The equilibrium beach profile is an important parameter in most oceanic numerical models. It is also required to predict the beach profile in designing coastal engineering projects (Bruun, 1954; Dean, 1977). Although the concept of equilibrium does not represent the true profile, it is a practical tool for beach profile analysis (Mangor 2004). Mathematical simulation of the seabed (equilibrium beach profile) has been widely investigated (Özkan-Haller and Brundidge 2007; Holman *et al*. 2014; Lopez *et al*. 2016; Różyński 2021; Faraoni 2019 and 2020; Maldonado 2020). The best mathematical model for simulating an equilibrium profile was introduced by Dean (1977) because of his extensive field research. According to the profiles collected by Hayden (1975), Dean (1977) reported (Equation 1) that the shape of the equilibrium beach profile depends on the size of the sediment grains. He assumed that the equilibrium in the beach profile is directly related to the loss of wave energy per water volume unit commonly used in applications (Dean, 1991).

$$h = Ax^{2/3} \tag{1}$$

where, h and x represent the water depth and the distance from the coastline (m), respectively. A is also a dimensional constant (scale parameter) depending on the size of the sediment grains and their fall rate. Experimental correlations between the scale parameter and the size of sediment grains or their fall rate make it possible to calculate the equilibrium beach profiles (Aragonés *et al*., 2016; Dean, 1991). In other studies, Lorson (1991) further developed the equilibrium profile of Dean (1977) by measuring the grain size changes along with the profile in Equation 2.

$$h = A_*[x + \frac{1}{\lambda}(\frac{D_0}{D_\infty} - 1)(1 - e^{-\lambda x})]^{2/3} \tag{2}$$

where, A_* represents the scale parameter ($m^{1/3}$), D_0 is the wave energy dissipation per unit volume near the coastline, D_∞ represents the wave energy dissipation per unit volume ($Nm/m^3/s$), and λ is the characteristic length describing the degree of proximity of D_0 to D_∞. To simulate the equilibrium profiles and describe the beach profiles on the beaches of the Mediterranean Sea, Komar and McDougal (1994) introduced an exponential relation as Equation 3.

$$h = \frac{S_0}{k}(1 - e^{-kx}) \tag{3}$$

where, S_0 is the slope of the beachfront and a function of the sediment grain size and wave parameters. k is an adjustable coefficient to determine the concavity of the beach profile, which is approximately a function of the parameters of the beachfront slope (S_0) and the depth of the bed (h_c). In other studies, Sierra *et al*. (1994) compared the field data from the Catalan Coast and found that it is the most appropriate mathematical model for describing the seabed (Equation 4).

$$h = x/(A + Bx) \tag{4}$$

where, x is the distance from the coastline. A and B are the experimental parameters. According to the proposed mathematical models and their accuracy in simulating the beach profiles, to extract and increase the accuracy of the mathematical model, the interpolation method can be used along with the estimation of a polynomial equation. One of the easiest ways to calculate the depths from the coastline ($f(x)$) is to estimate a polynomial equation such as *p(x)* of the *n*th degree in *x*. The polynomial equation *p(x)* must be calculated in a way that its value at x_i shows the depth from the coastline (f_i) (Equation 5).

$$P(x_i) = f_i \qquad i = 0,1,\cdots,n \tag{5}$$

Finally, it can be said that there is only one algebraic polynomial *P* of degree n in Equation 5 representing the continuous function of *y= f(x)* (Gill *et al*., 2021). The polynomial models provide an accurate description of bed changes to study the pattern of erosion and sedimentation in different areas according to their profile. Moreover, they are simple and do not require complex parameters, while Dean's model require more complex and multiple parameters for calculations. The solutions discussed above describe the desirable characteristics for the average profile over a long period and can also be useful for investigating the long-term changes in the volume of marine sediments. Research in the field of coastal engineering analyzes and studies the waves, longitudinal and transverse marine currents, longitudinal and transverse sediment transport, longitudinal and transverse profiles, as well as coastline changes. To date, studies on beach profiles (longitudinal and transverse) have received more attention in countries with maritime boundaries. Countries such as the United States, the Netherlands, the United Kingdom, Australia, and Norway have been among the pioneers in this field. In the present research, the transverse profiles of the southern beaches of the Caspian Sea (Mazandaran Coast) were investigated. Although the beaches are constantly changing, it is important to achieve a mathematical relationship mostly consistent with the beach profile of the study area. According to the high costs of field measurements, bed changes can be estimated over a period using mathematical equations, which can be widely used in coastal models. In some study areas, mathematical models, e.g., Dean's and, Komar-McDougal's models are far from the real profile measured in this research, Accordingly, the polynomial models were used in this research. Furthermore, because of the low tide of the Caspian Sea (Zereshkian and Mansoury 2020; Fallah and Mansoury 2022) and the widespread use of its beaches by tourists, it is necessary to study the changes in the bed and the equilibrium beach profile to identify the safe areas for swimming and predict the suitable marine areas for coastal and offshore structures.

2. Materials and Methods

Investigating the changes in the coastlines and predicting their changes in the future, plays an important role in determining the boundaries of the coastal cities, creating the constructions related to these cities, and determining the location of ports. In this regard, in the present study, the profiles of the southern beaches of the Caspian Sea (Mazandaran coast) were investigated, in different stations e.g., Amirabad, Larim, Babolsar, Fereydonkenar,Mahmoudabad, Sisangan, Nowshahr, and Namakabroud sandy coastal areas in Mazandaran Province (Figure 1).

Figure 1. The study areas on the southern coast of the Caspian Sea

In the present research, to extract a suitable mathematical model for the bed of the study areas, two common mathematical models, Dean's (1977) and, Komar-McDougal's (1994) models, were used. Then, the direct polynomial interpolation method was applied to obtain a suitable polynomial mathematical model for the bed of the study areas. To investigate the beach profiles of the study areas, the data obtained from the field measurements which include the seasonal depth changes in Amirabad, Larim, Fereydonkenar, Mahmoudabad, Sisangan, and Namakabroud (Table 1) for the summer and winter of 2013. Following that, the average diameters of the sediments and the significant wave heights (Table 2) obtained from the maritime organizations (Ports and Maritime Organization and the National Research Center of Caspian Sea) were used to model the bed. Then, to determine the beach profile, the field measurements were performed in Babolsar and Nowshahr between 2018 and 2019. The measurements were performed in autumn, winter, spring, and summer. To measure the water depth, in the marine section, several stations were considered at certain distances (near the beach with a higher resolution). The coordinates of the desired stations were extracted from Google Earth and recorded in GPS. Finally, at each station, depth measurements were performed by Echo Sounder. Also, to determine the median diameter of the sediment (D50), the sediment samples were taken by Van Veen Grab at depths of 0, 0.5, 1.5, 3, 6, and 10 meters. After the granulation process by Shaker, Gradistat Software was used to obtain the parameter of the median diameter of the sediment (D50) (Blott and Pye 2001). According to the measured data (bathymetry), the transverse profile and equilibrium profile were extracted in each study area.

Table 1. The geographical location of beach profiles (National Center for Caspian Sea Studies and Research 2013)

Stations	Latitude (°N)	Longitude (°E)
Amirabad	36.847202	53.315617
Larim	36.769833	52.956055
Babolsar	36.71526	52.67731
Fereydonkenar	36.691077	52.524987
Mahmudabad	36.649654	52.315960
Sisangan	36.581952	51.820675
Nowshahr	36.66178	51.49452
Namakabroud	36.695658	51.297986

According to Dean’s (1977) and, Komar-McDougal’s (1994) models and by determining their experimental coefficients, the profiles obtained from these two mathematical models were compared to the equilibrium beach profiles of the study areas.

Table 2. Mean significant wave heights (Hs) (https://www. pmo.ir 2012-2013 and 2017-2018)

Stations	$\overline{H}_s\ (m)$			
	Winter	Spring	Summer	Autumn
	2012-2013			
Amirabad		0.594	-	0.666
Larim	0.690	-	0.713	-
Fereydonkenar	0.782	-	0.750	-
Mahmudabad	0.726	-	0.777	-
Sisangan	0.814	-	0.775	-
Namakabroud	0.628	-	0.616	-
	2017-2018			
Babolsar	0.771	0.681	0.743	0.859
Nowshahr	0.791	0.760	0.741	0.707

Percentage error between equilibrium profiles and mathematical models in research areas is calculated from Equation 6 (Bodge, 1992; Etemadi *et al*, 2021):

$$\text{RRMSE} = \left(\frac{1}{N}\sum_{1}^{N}\frac{(h_1-h_2)^2}{{h_1}^2}\right)\times 100 \qquad (6)$$

where, RRMSE is the average of relative root mean square errors; h_1 is the depth from the equilibrium beach profile, that obtained from the measured data; h_2 is from the mathematical models, and N is number of positions measured or number of data points in each transect. To calculate the polynomial mathematical model and compare it to Dean's (1977) and, Komar-McDougal's (1994) models, the polynomial interpolation method was used. In this method, it is first assumed that a polynomial with a degree of n is the answer to the problem (Nikookar and Darvishi, 2013). These polynomials are then identified according to Equation 7:

$$P(x) = b_0 + b_1 x^1 + b_2 x^2 \ldots b_n x^n \tag{7}$$

The above function must be implemented for all points $[x_0, f(x_0)], [x_1, f(x_1)], [x_2, f(x_2)], \ldots, [x_n, f(x_n)]$. Then, by placing the desired points in Equation 7, the system of linear equations is formed (Equation 8).

$$\begin{cases} x = x_0 \Rightarrow b_0 + b_1 x_0 + b_2 x_0^2 + \cdots + b_n x_0^n = f(x_0) \\ x = x_1 \Rightarrow b_0 + b_1 x_1 + b_2 x_1^2 + \cdots + b_n x_1^n = f(x_1) \\ \vdots \\ x = x_n \Rightarrow b_0 + b_1 x_n + b_2 x_n^2 + \cdots + b_n x_n^n = f(x_n) \end{cases} \tag{8}$$

Then, a matrix form $Ax = B$ is used to solve the system of the linear equations. *A* is a square matrix of order n with known components, *B* is a column vector with n known components, and *x* is a column vector with n unknown components (Equation 9).

$$A = \begin{bmatrix} 1 & x_0 & x_0^2 \cdots & x_0^n \\ 1 & x_1 & x_1^2 \cdots & x_1^n \\ \vdots & \vdots & \vdots \;\cdots & \vdots \\ 1 & x_n & x_n^2 \cdots & x_n^n \end{bmatrix}, \quad x = \begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_n \end{bmatrix}, \quad B = \begin{bmatrix} f(x_0) \\ f(x_1) \\ \vdots \\ f(x_n) \end{bmatrix} \tag{9}$$

There are several methods for calculating an n-equations n-unknowns' system. One of the common methods for solving a $Ax = B$ system is to use the inverse form of matrix *A* (Equation 10).

$$A^{-1}Ax = A^{-1}B \Rightarrow x = A^{-1}B \tag{10}$$

Finally, the solution of the matrix equation is obtained by $x = A^{-1}B$ (Gill *et al*. 2021). In the present study, the inverse matrix method was used to solve the system and extract the unknown coefficients of polynomial equations. Based on the volume of the available data, MATLAB software was used to calculate the system of equations.

3. Results and Discussion

3.1. Granulation of bed sediments

Using the output data of Gradistat software, the average percentages of the bed sediments during the four seasons at the desired depths were calculated (Figure 2). With increasing the depth, the percentage of fine-grained sediments, including very fine sand and clay were increased, while coarse and medium sands had a decreasing trend. Thus, the smaller sediments are transported away from the coast and only larger diameter sediments remain nearshore (Dean and Dalrymple 2004).

According to the calculations of the type and percentage of the sediments, the average percentage of the sediments in autumn, winter, spring, and summer were calculated that have been represented in Table 3.

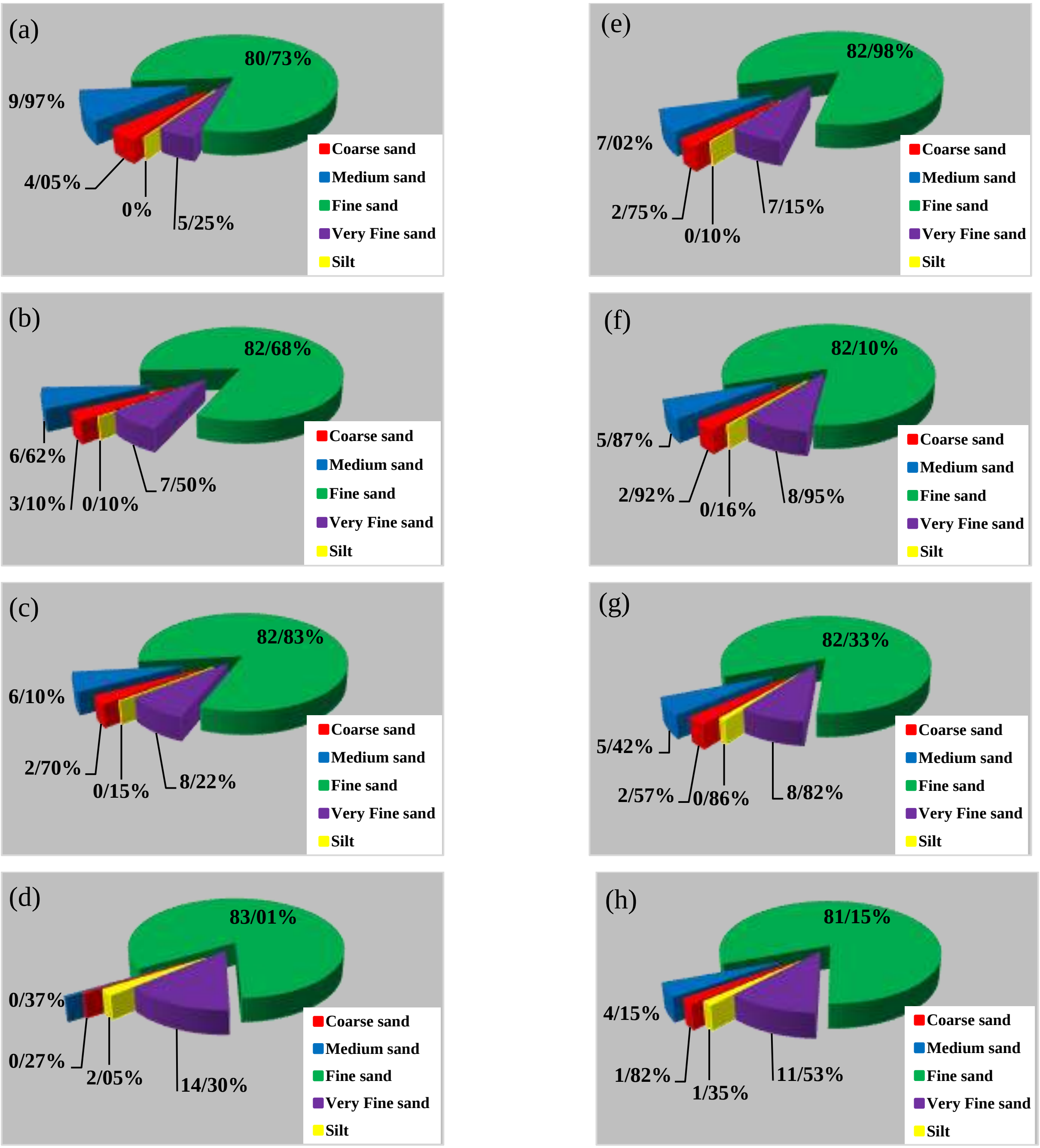

Figure 2. Comparison of the average percentage of bed sediments at various depths in Babolsar (a: 0.5 m) (b: 1.5 m) (c: 3 m) (d: 5 m) and Nowshahr (e: 0.5 m) (f: 1.5) (g: 3 m) (h: 5 m) (Safari *et al.*, 2022)

Table 3. The average percentage of sediments in autumn, winter, spring, and summer (Safari *et al*., 2022)

		Average				
		Coarse sand (%)	Medium sand (%)	Fine sand (%)	Very Fine sand (%)	Silt (%)
Babolsar	Autumn	1.8	4.3	82.7	11	0.36
	Winter	2.97	3.7	83.75	9.9	1.6
	Spring	2.25	3.67	88.85	4.85	1.5
	Summer	4.1	11.4	74.27	9.35	1.7
Nowshahr	Autumn	2.55	4.75	75.47	16.22	1
	Winter	2.2	4.97	88.82	3.67	1.2
	Spring	1.65	3.05	92.75	2.15	0.5
	Summer	3.67	9.67	71.45	14.42	0.5

In both east and west stations, the predominant type of bed sediments was fine sand and the least frequent type of bed sediments was silt. According to the data related to the significant wave heights, the closure depths were calculated in Babolsar and Nowshahr. Then, to investigate the changes in the beach profiles in the desired stations, the mean (equilibrium) profile was extracted from the maritime organizations (Ports and Maritime Organization, and the National Research Center of Caspian Sea) in different seasons according to the measured data and the closure depth (www.Pmo.ir, and www.wri.ac.ir).

3.2. Changes in bed slope and median diameter of sediment

In winter, due to the turbulence caused by the waves breaking, the sediments in the coastal areas are eroded and transported offshore by marine currents. Therefore, as a result of interference with the sediments transferred offshore, mounds are formed in the breaking zone. Finally, the slope of the bed follows a decreasing trend. In summer, due to the reduced wave height and energy caused by breaking, the sediments settle in the coastal areas and cause the formation of mounds and platforms in the area. Ultimately, the slope of the bed increases in summer (Matinfar, 2009). According to the beach profiles in Larim, Babolsar, Fereydonkenar, Mahmoudabad, Sisangan, Nowshahr, and Namakabroud stations, the slope of the bed in summer had an increasing trend compared to the winter (Figure 3).

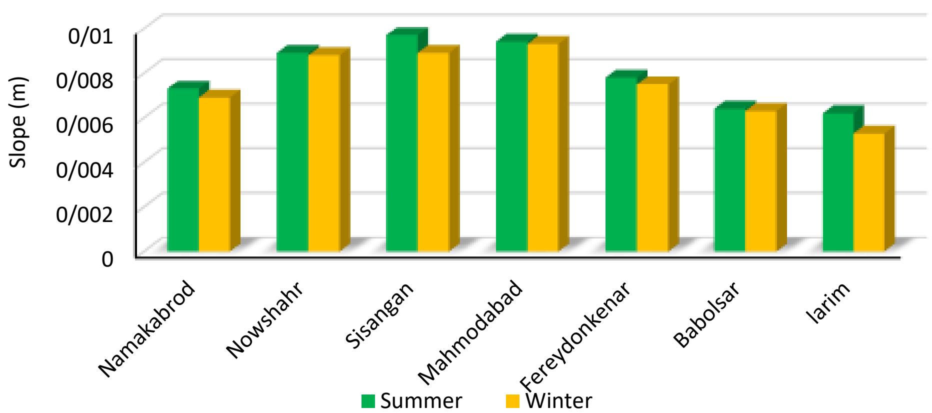

Figure 3. Bed slope changes in the studied coastal areas in summer and winter

The average median diameter of the bed sediment particles on the southern beaches of the Caspian Sea decreased from west to east (Figure 4). By increasing the percentage of the fine-grained sediments and the consequent increase of the deposits, the bed slope in the eastern areas decreased compared to the western ones. In the following table, the coefficients required to model the seabed were calculated by the mathematical models' equations are represented (Table 4). Dean's model depends on the scale parameter, and the scale parameter relies on falling speed and the median diameter of the sediment. By changing the median diameter of the sediment (D50), the scale parameter (A) will also change. Moreover, in Komar-McDougal's model, the amount of concavity (k) depends on the slope of the beachfront (S_0). These values have been calculated for research areas, e.g., Babolsar and Nowshahr (Figure 5).

Table 4. Parameters calculated for mathematical modeling of Babolsar (Ba) and Nowshahr (No) equilibrium profiles

Calculated parameters	2017-2018							
	Autumn		winter		Spring		Summer	
	Ba	No	Ba	No	Ba	No	Ba	No
Closure depth (m)	5.79	5.33	5.20	5.13	4.59	5	5.01	4.77
Beachfront slope	0.05	0.05	0.04	0.03	0.03	0.04	0.04	0.02
Amount of concavity (1/m)	0.008	0.0093	0.0076	0.0058	0.0065	0.008	0.0079	0.004
The median diameter of sediment (mm)	0.151	0.191	0.173	0.192	0.182	0.196	0.171	0.184
Sediment particles fall rate(m/s)	0.0164	0.0239	0.0205	0.0241	0.0222	0.0249	0.0201	0.0226
Scale parameter ($m^{1/3}$)	0.0681	0.0875	0.0790	0.880	0.0833	0.0899	0.0779	0.0843

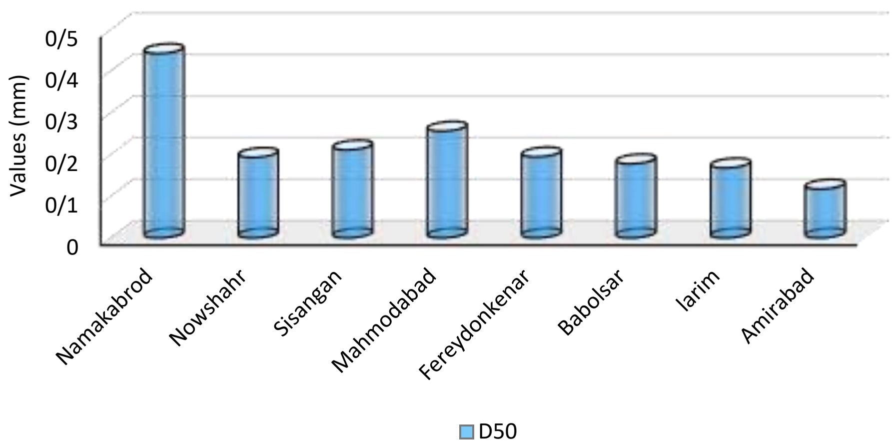

Figure 4. Comparison of the average median diameters of sediments from east to west in the studied coastal areas

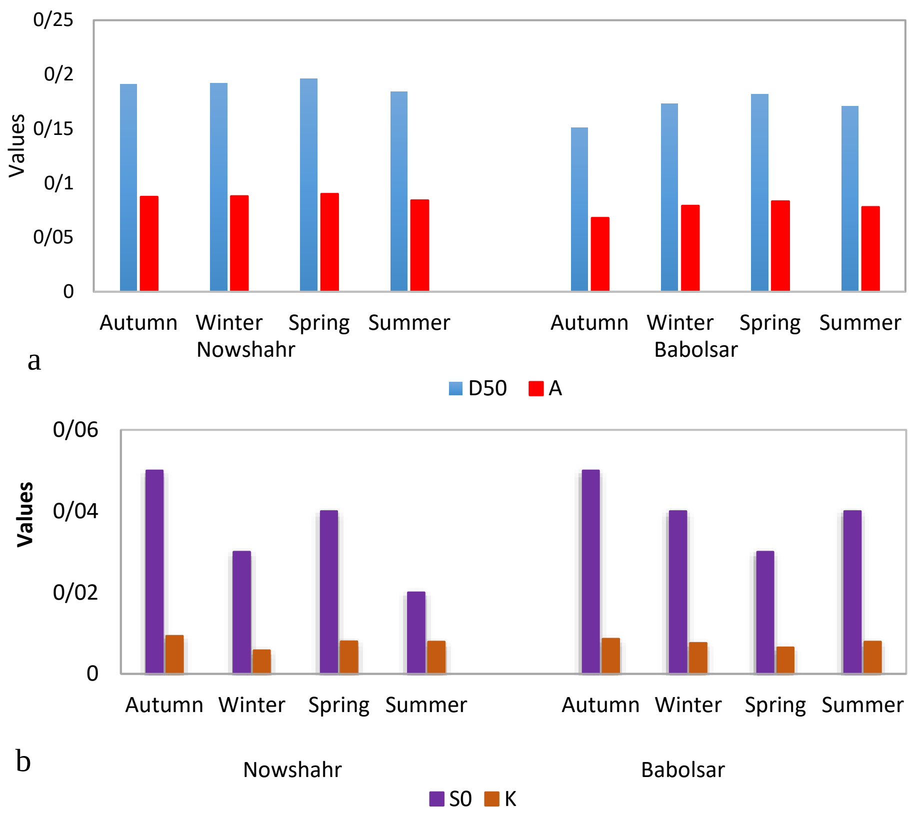

Figure 5. Comparison of the median diameters of sediments, sediment scale parameters (a), beachfront slopes, and the amounts of concavity (b) in the studied coastal areas

3.3. Equilibrium beach profile of the study areas

The eroded sections were identified on the equilibrium profile (Figure 6). Considering the beach profile of Amirabad, at a distance of 100 to 150 m from the coastline, sedimentation and erosion occurred in spring and autumn based on the equilibrium profile. From a distance of 200 m to the deeper areas, the beach profiles were aligned with the equilibrium profile. At a distance of 20 to 60 m, there was a platform at a depth of 1 m. The slope of the bed from the end of the platform to a distance of 160 m is equal to 0.025 (Figure 6a). In the coastal region of Larim, in summer, erosion occurred at an average distance of 200 to 350 m from the coastline. These eroded sediments were transported to the coastal areas by relatively weak waves, however, sedimentation occurred at the distances of 70 to 100 m and 120 to 190 m from the coastline. In winter, due to the erosion and transfer of the sediments offshore, an increase in depth was observed. At a distance of 120 to 190 m, the beach profile was eroded in winter and the eroded sediments were deposited at a distance of 200 to 350 m. The equilibrium profile showed a platform at a distance of 40 m from the beach at a depth of 0.5 m. Two mounds were formed at distances of 100 and 200 m (Figure 6b). On the coast of Fereydonkenar, in summer, at the distances of 110 to 200 and 250 to 350 m from the coastline, the seabed was eroded. In winter, from the distance of 130 to 200 m and 80 to 110 m, erosion and sedimentation occurred, respectively. Also, at a distance of 240 m, a mound was formed at a depth of 2 m (Figure 6c). The beach bed of Mahmoudabad was also eroded in summer at distances of 70 to 160 m and 200 to 400 m from the coastline. In winter, due to the transfer of the sediments to the sea and the sedimentation process, the depth changes compared to the equilibrium profile are reduced at distances of 70 to 160 m and 200 to 370 m. The equilibrium profile also had a platform at a depth of 4 m with a length of 150 m (Figure 6d). In the coastal area of Sisangan, the beach profile was eroded in summer at a distance of 250 to 310 m from the coast, leading to an increase in the depth. These eroded sediments were gradually transported by low-energy waves to areas near the coast. In winter, due to the increase in the wave height, the sediments were eroded from the area near the beach and moved to areas far from the beach (up to 250 m from the coastline). At a distance of 350 m, a mound with a height of 1.5 m was formed (Figure 6e). In summer, at distances of 190 to 300 m and 100 to 190 m, erosion and sedimentation occurred to the equilibrium profile on Namakabroud beach, respectively. In winter, due to the erosion at a distance of 50 to 70 m, the depth increased, and at the distance of 200 to 300 m, a mound was formed due to the sedimentation (Figure 6f). In summer, at distances of 190 to 300 m and 100 to 190 m, erosion and sedimentation occurred relative to the equilibrium profile on Namakabroud beach. In winter, due to erosion, at a distance of 50 to 70 m, the depth increased relative to the equilibrium profile, and at a distance of 200 to 300 m, a mound was formed due to sedimentation (Figure 6f).

On the Babolsar coast, according to the measurements made in autumn, winter, spring, and summer (Figure 7a), at distances of 100 to 200 m from the coastline, the beach profile in autumn included a platform and a sedimentary mound. In winter, due to the erosion and transfer of the sediments away from the beach, a platform of sediment deposition was formed at a distance of 300 to 400 m. In spring, at a distance of 100 to 210 m, erosion occurred relative to the equilibrium profile. Finally, in summer, at a distance of 250 to 600 m from the coastline, erosion occurred compared to the equilibrium profile. At the distances of 100 and 190 m, mounds were created by the deposition of sediments transferred to these areas. Most of the bed changes during the four seasons occurred under the direct influence of the waves on the bed.

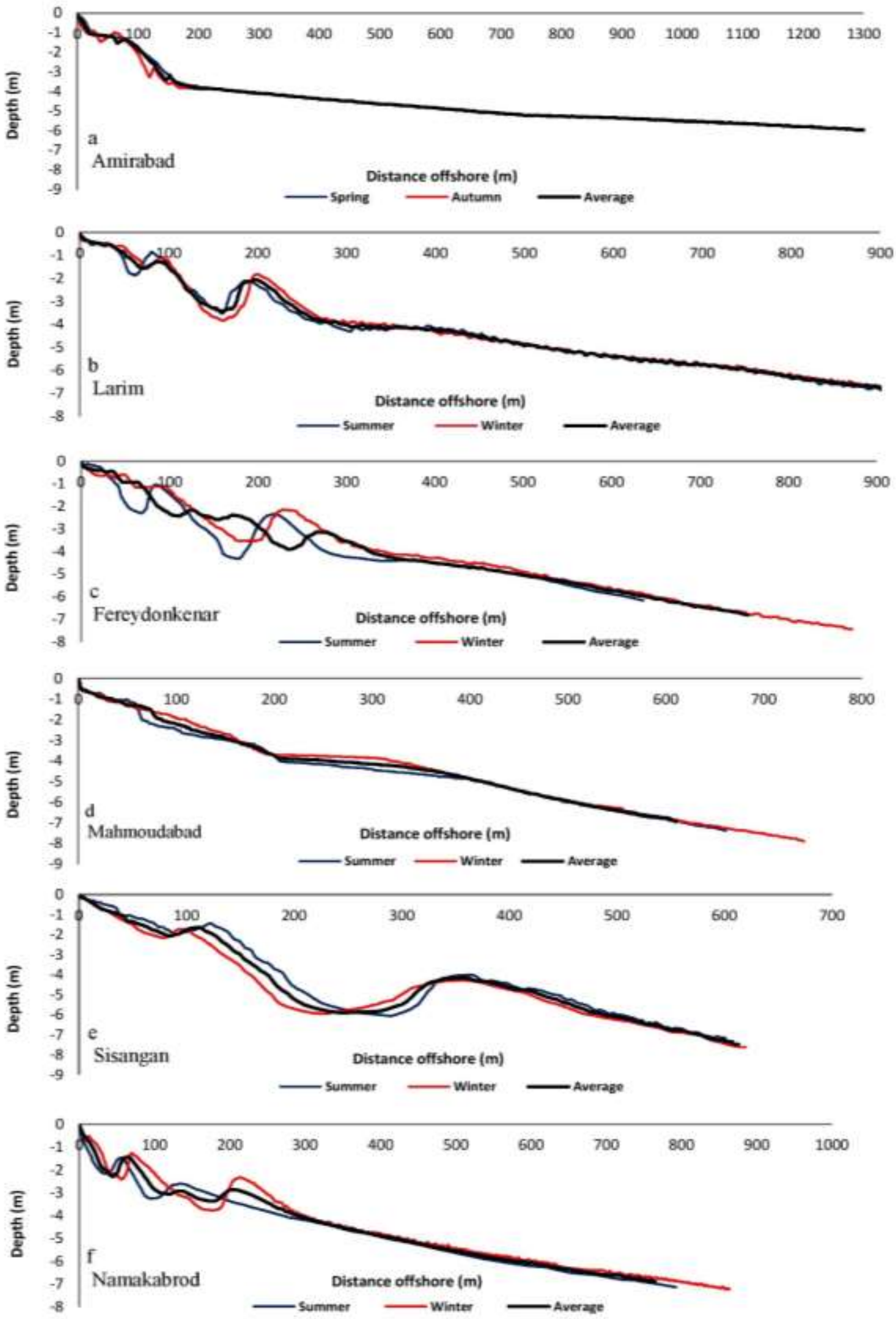

Figure 6. Changes in beach profile compared to the equilibrium profile in research areas

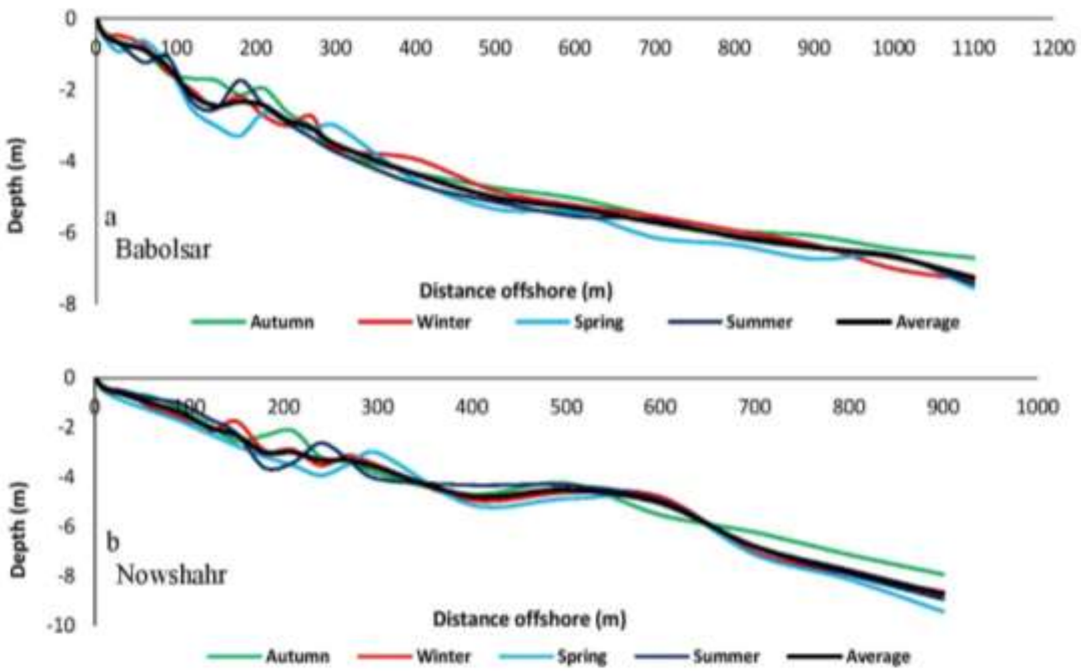

Figure 7. Changes in beach profile compared to the equilibrium profile in a) Babolsar and b) Nowshahr

The equilibrium profile also had a platform at a distance of 30 to 60 m and a depth of 0.5 m. At a distance of 60 to 150 m, the profile had a slope equal to 0.018. A mound was also formed at a distance of 200 m. In autumn, up to a distance of 120 m from Nowshahr beach and due to the mound created at a distance of 200 m, the depth decreased relative to the equilibrium profile. Beach profiles were eroded in winter from a distance of 50 to 120 m relative to the equilibrium profile. In spring, due to the erosion near the beach, the depth increased compared to the equilibrium profile. Finally, in summer, due to the transfer of the sediments to the areas near the beach, the depth at a distance of 150 m decreased to zero. The equilibrium profile had a slope of 0.014 from the coastline to a distance of 150 m and a platform at a distance of 190 to 210 m from the beach. This profile was considered an indicator to measure the rate of erosion and sedimentation during different seasons. The slope of the profile at a distance of 290 to 400 m was equal to 0.011. A 150 m-long platform was formed at a distance of 400 to 550 m from the beach (Figure 7b).

3.4. Mathematical modeling of the equilibrium beach profile

The beach profiles of the study areas were plotted using the existing data and field measurement data. Considering the parameters such as the median diameter of the sediments and the significant wave heights, the coefficients in Dean’s (1977) model or Komar-McDougal’s (1994) model were calculated for the mathematical modeling of the bed (Figure 8). According to the results, Dean's (1977) mathematical model was more consistent with the profile of the Caspian coast. Based on the profiles, on average, moving from the coastal areas to the offshore areas, the bed slope had an increasing trend, causing the equilibrium profile of these regions to be more consistent with Dean's (1977) model compared to Komar-McDougal’s (1994) model.

According to Figure 8, the seabed changes were mainly located between Dean's and Komar-MacDougal's models. Thus, these changes, from the east coast of Mazandaran to its west coast, are more consistent with the Komar-MacDougal and Dean's models, respectively. On Amirabad beach, for mathematical modeling of the bed, the equilibrium profile was simulated by Dean's and Komar-McDougal's models (Figure 8a). The equilibrium profiles obtained from the mathematical models were compared to the equilibrium profiles obtained from the measured data. The profiles obtained from the two mathematical models in question were in good agreement with the equilibrium profile, up to a distance of 100 m from the coastline. From a distance of 100 m to the deeper areas, the exponential model had the most agreement with the equilibrium profile of the area compared to Dean's model. In Dean's model, the differences in the type and size of the bed sediments caused errors in the calculation of the equilibrium profiles. In Amirabad, according to the size of the median diameter of the sediments, the equilibrium profile obtained from Dean's model did not correspond much to the research area. Next, on the beach of Larim, the equilibrium profile obtained from Dean's model had good agreement with the equilibrium profile.

The equilibrium profiles obtained from the exponential model were lower than the equilibrium profile due to the increased slope near the beach (Figure 8b). On Fereydonkenar beach, the equilibrium profile from the coastline to the deep areas was most consistent with Dean's equilibrium profile compared to the exponential profile (Figure 8c). In Mahmoudabad beach, for mathematical modeling, Dean's equilibrium profile was most consistent with the equilibrium profile from the coastline offshore (Figure 8d). However, the exponential profile, from a distance of 200 to 400 m, was compatible with the equilibrium profile (Figure 8d). On Sisangan beach, Dean's model was in good agreement with the equilibrium profile, while the exponential model was consistent with the equilibrium profile at a distance of 190 to 440 m (Figure 8e). Seabed changes depend on various factors such as the type and granularity of sediments, sea current, and wave height (wave breaking) on the nearshore. Some research areas (i.g., Sisangan), in the areas near the coast, have a mixed bed (pebbles and sand), so Hydrodynamic interaction with the seabed can cause relatively drastic changes in the seabed in sandy areas (Figure 8e).

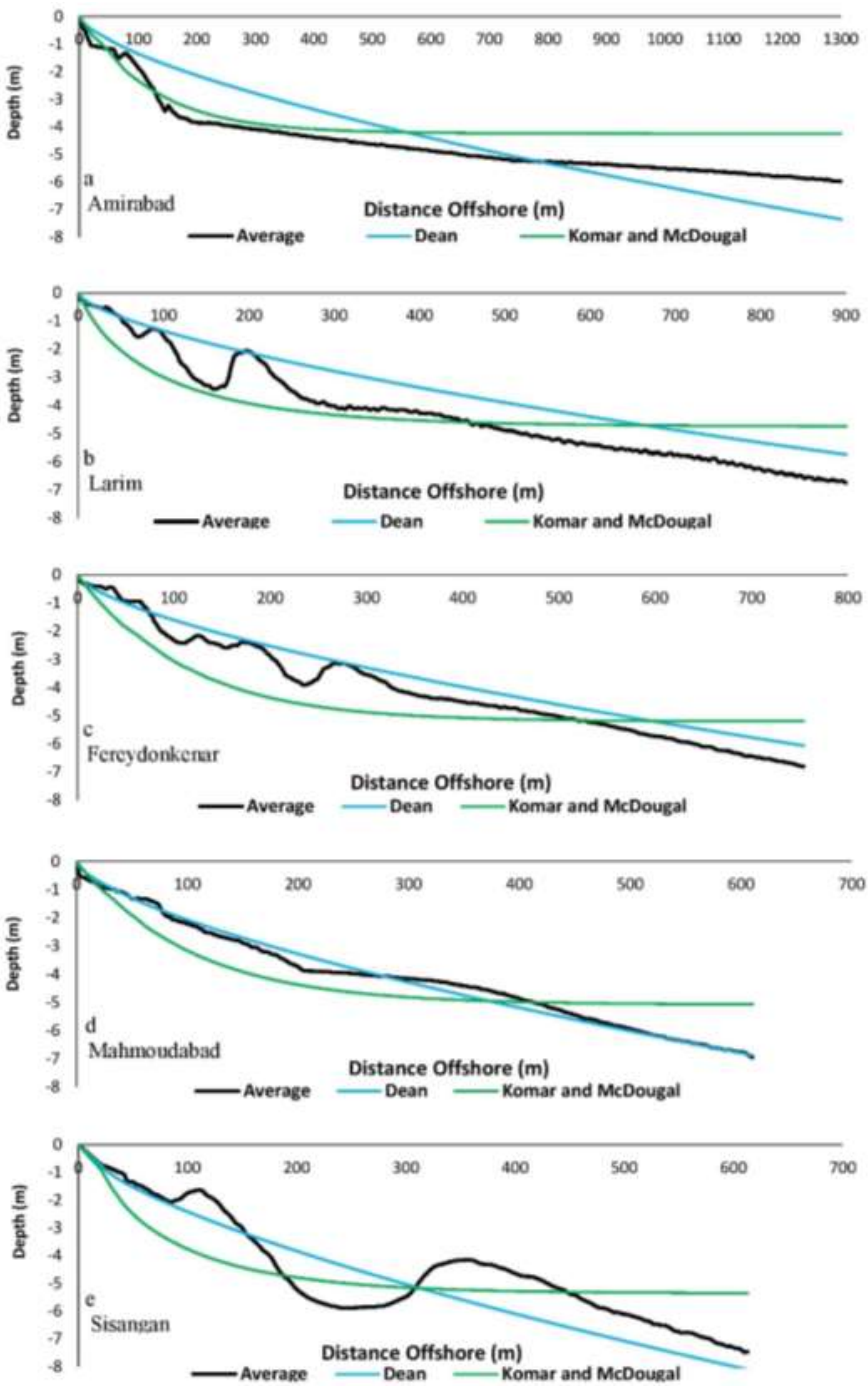

Figure 8. Simulation of equilibrium profiles by Dean’s and Komar-McDougal’s model

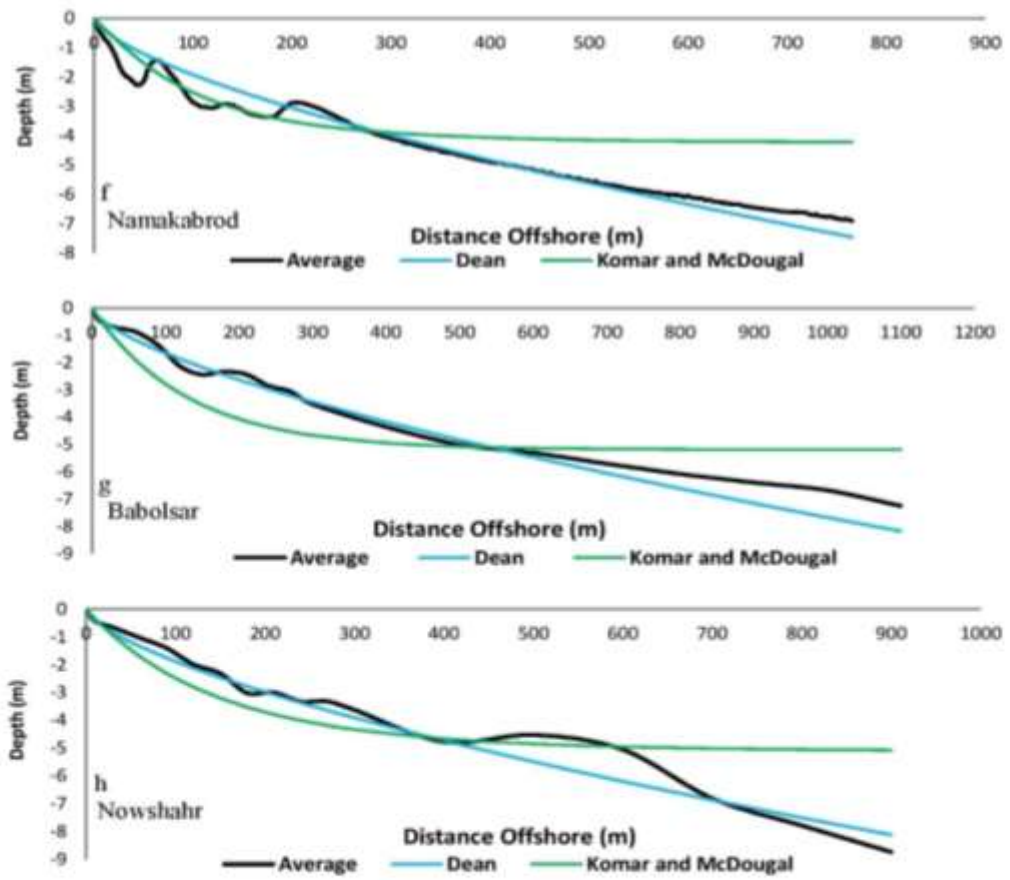

Figure 8. Continued

On Namakabroud beach, the equilibrium profile obtained from Dean's model and the exponential mathematical models showed a good agreement with the equilibrium profile from the coastline to a distance of 250 m. From the depth of 250 m to the deeper areas, Dean's profile had a good agreement with the equilibrium profile (Figure 8f). In Babolsar beach, the equilibrium profile obtained from Dean's model was more in line with the equilibrium profile compared to the exponential model. Dean's model matched the equilibrium profile up to 550 m from the coastline. However, the exponential model did not have a good agreement with the equilibrium profile due to the depression in the coastline area up to a distance of 500 m (Figure 8g). On the beach of Nowshahr, the equilibrium profile obtained from Dean's model from the coastline to the deep areas was most consistent with the equilibrium profile. However, the exponential mathematical model at a distance of 400 to 600 m from the beach had good agreement with the equilibrium profile (Figure 8h).

3.5. Error of mathematical models

Finally, according to the studies performed in mathematical model for the bed of the southern beaches of the Caspian Sea (Table 5), the results indicate that the equilibrium profile obtained from Dean's model, except for the coastal region of Amirabad, had the lowest error and the highest correlation with the mean (equilibrium) profiles compared to the other mathematical models.

Table 5. Percentage error between equilibrium profiles and mathematical models in research areas

Beach profile	Dean’s model RRMSE (%)	Komar-McDougal’s model RRMSE (%)
Amirabad	8.38	7.59
Larim	3.67	5.57
Fereydonkenar	1.36	5.21
Mahmoudabad	0.24	3.69
Sisangan	4.13	5.69
Namakabroud	0.84	7.63
Babolsar	0.99	7.61
Nowshahr	1.02	9.31

3.6. Polynomial modeling of equilibrium beach profile

To derive the polynomial equations, the polynomial interpolation method was used. The polynomial mathematical models were matched to the equilibrium profiles obtained from the measured data (Figure 9). This study aimed to study and accurately describe the beach profiles mathematically. Differences in the coefficients and order of the equations indicate differences in the behavior and shape of the beach profiles in different regions. The beach profiles in various regions were different owing to the differences in the topography and slope of the bed, the type, and percentage of the bed sediment grains, as well as the process of erosion and sedimentation during different seasons and periods. These factors caused a different profile for each region from its neighboring regions, followed by different mathematical models for description.

According to the studies, the slope of the bed decreased from west to east, affecting the transport and displacement of the bed sediments. Areas with a gentler slope than the other coastal areas were more exposed to the erosion and sedimentation processes. Ultimately, a polynomial mathematical model is proposed to describe it.

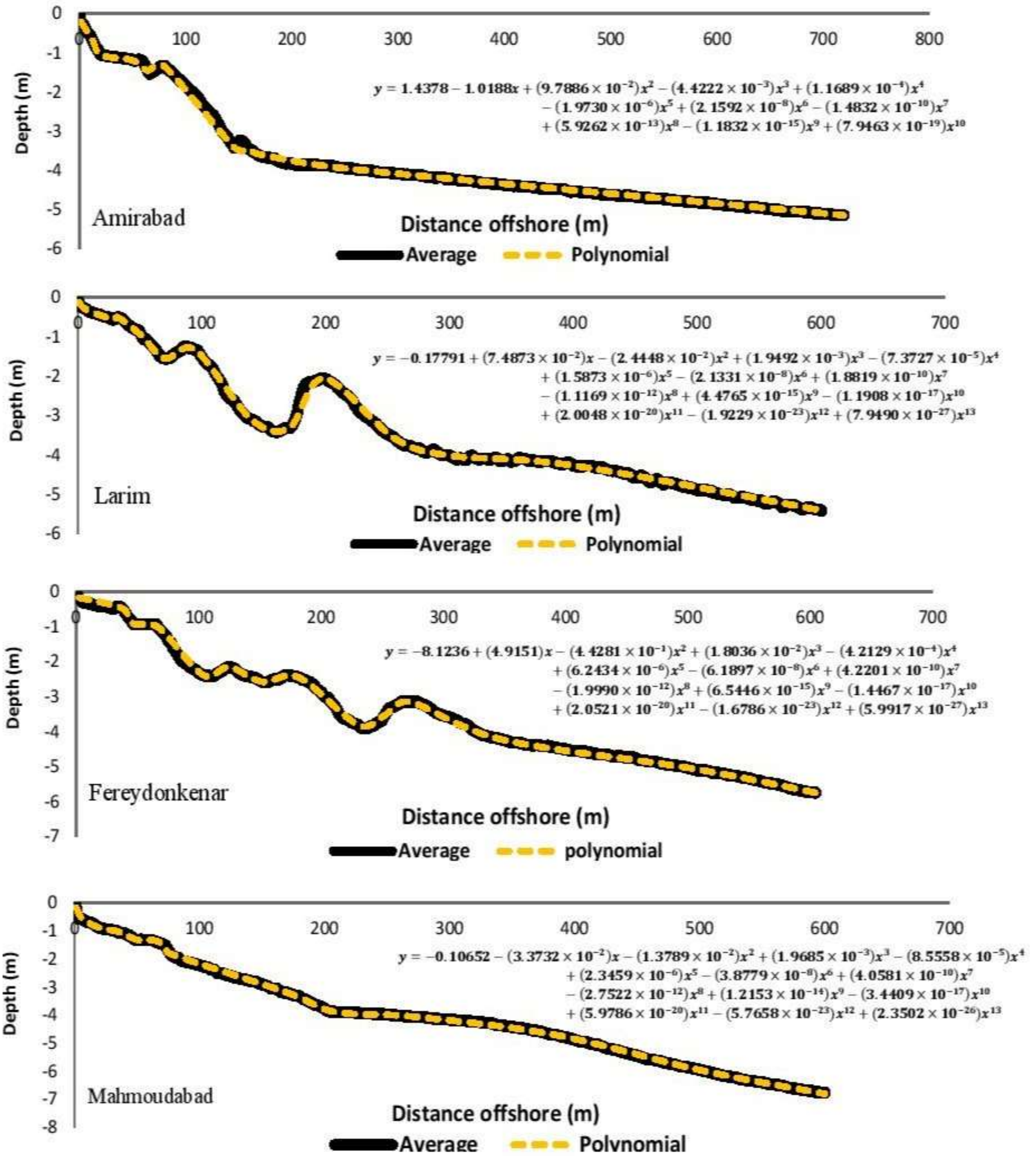

Figure 9. The polynomial models along with equilibrium and Polynomial profiles of research areas

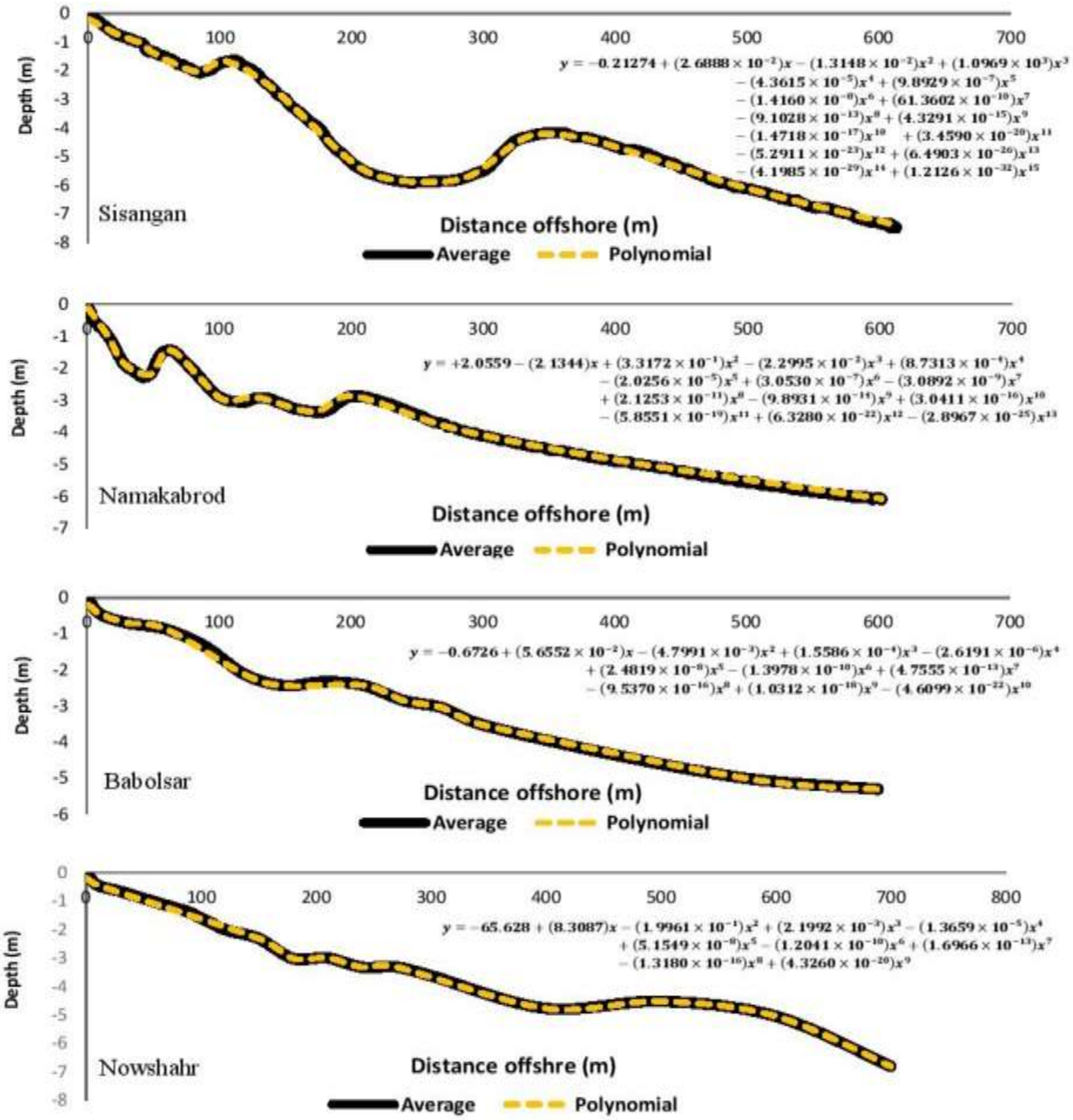

Figure 9. Continued

Conclusion

In this study, the transverse profiles of the Mazandaran coasts in the southern half of the Caspian Sea were studied. In this regard, the equilibrium beach profiles of the research areas were compared to Dean's model and Komar-McDougal’s model (1994), as well as the polynomial models based on the field measurements. According to the studies performed to mathematically model, the bed of the southern beaches of the Caspian Sea in Mazandaran province, the equilibrium profile obtained from Dean’s model was most reliable with the mean (equilibrium) profiles of research areas, except for the coastal region of Amirabad. In autumn, winter, spring, and summer (2018 to 2019), on average, the most bed changes were observed from the coastline to a distance of about 400m. In these areas, the waves had the greatest impact on the bed and caused the transfer of the sediments, erosion, and sedimentation. Dean's equilibrium equation was more consistent with the sandy beaches of Babolsar and Nowshahr compared to Komar-McDougal's model. However, considering the large initial curvature in the profile and the creation of the abdominal area in the shallow area, it was below the normal profile in most areas, leading to more significant errors compared to Dean's equation. Dean's model predicted the best equilibrium profile for the sandy beaches of the studied areas by applying the parameter of the median diameter of the bed particles in the proposed equation. Compared to Dean's and Komar-McDougal's models polynomial equations described the beach profiles of the research areas during different periods more accurately.

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Data availability

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